AIChE WILEY InterScience DISCOVER SOMETHING GREAT

Stability Analysis of Core-Annular Flow and Neutral Stability Wave Number

Oscar M. H. Rodriguez
Dept. of Mechanical Engineering, Engineering School of Sao Carlos, University of Sao Paulo, 13566-970, Sao Carlos, SP, Brazil

Antonio C. Bannwart
Dept. of Petroleum Engineering, Faculty of Mechanical Engineering, The State University of Campinas, 13083-970, Campinas, SP, Brazil

DOI 10.1002/aic.11361
Published online November 13, 2007 in Wiley InterScience (www.interscience.wiley.com).

A general transition criterion is proposed in order to locate the core-annular flow pattern in horizontal and vertical oil–water flows. It is based on a rigorous one-dimensional two-fluid model of liquid–liquid two-phase flow and considers the existence of critical interfacial wave numbers related to a non-negligible interfacial tension term to which the linear stability theory still applies. The viscous laminar–laminar flow problem is fully resolved and turbulence effects on the stability are analyzed through experimentally obtained shape factors. The proposed general transition criterion includes in its formulation the inviscid Kelvin-Helmholtz's discriminator. If a theoretical maximum wavelength is considered as a necessary condition for stability, a stability criterion in terms of the Eötvös number is achieved. Effects of interfacial tension, viscosity ratio, density difference, and shape factors on the stability of core-annular flow are analyzed in detail. The more complete modeling allowed for the analysis of the neutral-stability wave number and the results strongly suggest that the interfacial tension term plays an indispensable role in the correct prediction of the stable region of core-annular flow pattern. The incorporation of a theoretical minimum wavelength into the transition model produced significantly better results. The criterion predictions were compared with recent data from the literature and the agreement is encouraging.
 AIChE J, 54: 20–31, 2008
Keywords: liquid–liquid flow, oil–water flow, hydrodynamic stability, interfacial wave, core-annular flow

Introduction

The development of production and transportation pipelines for heavy oil, which is characterized by a viscosity higher than 10^2 cP and density close to water, can make the economic exploitation of early unprofitable oil reservoirs viable in the near future. Heavy oil reserves are estimated in 4.6 trillion barrels throughout the world and their importance tends to increase progressively, as light oil fields are exhausted.[1] The pipe flow of two-phase oil–water mixtures may form several spatial configurations or flow patterns of two phases. By analogy with gas–liquid mixtures, these can be grouped into three categories: dispersed flow, separated flow, and intermittent flow.[2] Dispersed flow includes oil bubbles in water, water drops in oil, and also water-in-oil and oil-in-water emulsions. Intermittent flows consist of relatively large oil bubbles separated by water slugs. Separated flows comprise core-annular flow (oil in the core, water in the

Correspondence concerning this article should be addressed to O. M. H. Rodriguez at oscarmhr@sc.usp.br.

annulus) and stratified flow patterns. Recently, the use of the core-annular flow pattern has been proposed as an attractive technological alternative to produce heavy oils in the Brazilian deep water production scenario by adding relatively small amounts of water, so as to lubricate the flow.[2–9]

The observation of all these flow patterns in a single apparatus depends on the fluid properties, pipe size, and geometry involved. For example, Charles et al.[10] performed equal density experiments with oil and water in a 2.54 cm I.D. and observed most of the above patterns, except stratified flow. Flores et al.[11] studied a mineral oil–water flow (density ratio 0.85 and viscosity ratio 20) in vertical and inclined 5 cm I.D. pipes, observing no annular flow pattern. Trallero[12] used a similar mineral oil and water in a 5 cm I.D. horizontal pipe, and did not see annular flow either. Elseth[13] studied a light oil–water flow in a 5.63 ID horizontal pipe and through more sophisticated experimental techniques was able to produce a more detailed and accurate flow pattern classification. This author did not see the annular flow either.

The interesting alternative for the artificial lifting of heavy oil is based upon the rather significant accumulated experience in the transport of highly viscous fluids by the injection of water, in such a way to create a continuous lubrication layer around the oil and to establish an annular liquid–liquid flow pattern, known as core-annular flow. The pioneer works of Russel and Charles[14] and Charles et al. for horizontal pipes showed that this flow pattern occurs when the two liquids have similar densities and relatively small quantities of water are added. Ooms,[15] Ooms et al.,[16] and Oliemans[17] showed that for the existence of the core-annular flow the annulus must be thin and there must be asymmetric interfacial waves. Joseph et al.[18] and Feng et al.[19] studied the hydrodynamic stability of two immiscible liquids flowing in a pipe, showing that the more viscous fluid must occupy most of the pipe's cross-section and that asymmetric interfacial waves might be responsible for the stabilization of the core phase. Howard and Patankar[20] studied the stability of a downward vertical core-annular flow and showed that, for short interfacial waves the interfacial tension plays an important role to stabilize the core. Bai,[21] Bai et al.,[22] and Bai and Joseph[23] carried out an experimental work and numerical simulations for upward-vertical core flow and presented results on two-phase pressure drop and interface shape. Rodriguez and Bannwart[8,9] offered detailed experimental data and proposed an analytical solution for the interfacial waves in upward-vertical core-annular flow.

Brauner and Maron[24] classified the liquid–liquid flow patterns in horizontal pipes according to the Eötvös number: at lower Eo interfacial tension effects are dominant and core annular flow will tend to occur, whereas at larger Eo the flow will tend to stratify instead. Bannwart[25] also proposed an Eötvös number-based criterion for the occurrence of core annular flow in a horizontal pipe. Barnea[26] and Barnea and Taitel[27] proposed the use of the viscous Kelvin-Helmholtz (VKH) and inviscid Kelvin-Helmholtz (IKH) analysis in order to identify two regions of instability for stratified gas–liquid flow: one region bounded between the VKH and IKH neutral stability lines, associated with the beginning of the instabilities, and the region "outside" the IKH neutral stability curve, which will result in either slug flow or annular flow. Trallero, based on a one-dimensional approach quite similar to the one carried out by the former authors, used the linear stability theory to deduce transition criteria for stratified liquid–liquid flow in horizontal pipes. This author achieved excellent agreement between the theoretical results and experimental data.

To understand why core-annular flow is a stable configuration, basically two classes of investigations have been led. Indeed, the most classical approach is that of hydrodynamic stability theory (Ooms[15] and Joseph et al.[18]). In this approach, very small disturbance waves are imposed to an initial given core-annular flow configuration, thus justifying the linearization of the equations describing the evolution of the disturbance with time or space. The analysis then focuses on which variables and flow conditions contribute to the disturbance decay i.e. stability of the initial configuration. In the case of horizontal core-annular flow, the density difference and the associated Archimedes force have, to the best of our knowledge, not yet been incorporated into the analysis, for the simple reason that the initially given configuration assumes a perfect circular and smooth interface between the two liquids which is unable to counterbalance Archimedes force. This configuration is known as "perfect core-annular flow" for which an exact solution can be found in the case of vertical (axi-symmetric) flow. In trying to find the mechanism that makes it possible for horizontal core-annular flow to be stable under gravity with density mismatch, Ooms et al.[16] and Oliemans[17] created the second approach for the investigation of core-annular flow stability, which is known as the levitation mechanism. Using arguments of the classical lubrication theory, they showed that an appropriately wavy core with relative motion between oil and water could generate a suitable pressure field around the viscous core and create a downward force able to counterbalance the buoyancy force on the core. In fact, simple visualization of core-annular flow shows that the interface is wavy rather than smooth. However, Joseph and coworkers[22] argued that in the annulus inertial effects would have a prominent role and proposed a different wave shape and stabilization mechanism called aerodynamic lift. In both proposed mechanisms, interfacial tension effects were discarded. Bannwart[25] proposed to focus on the effect of interfacial tension in modeling the near circular interface required by core-annular flow and the drag caused by the peripheral flow around the core. The combination of these two effects is able to counterbalance buoyancy (note that in heavy oil–water core-annular flow the Archimedes and capillary forces may be of the same order). In particular, if the interface is assumed to be smooth, such as used for the initial configuration in studies of hydrodynamic stability, it is possible to completely determine the interface shape and the correspondent average pressure difference between the core and annulus fluids as well. It is important to add that wettability phenomena play an essential role in the stabilization of horizontal core-annular flow. In fact, in order for the viscous core to be kept completely surrounded by water for a long time, the pipe wall must exhibit a hydrophilic–oleophobic behavior, i.e., interfacial tension forces must overcome oil-wall adhesion forces. This results in a zero-contact angle of the oil core relative to the wall. In fact, observations of horizontal oil–water core-annular flow in a glass pipe at low velocity by the present authors (Bannwart et al.[2]] indicate that between the oil core and the top wall there is a very thin but continuous water layer which allows the oil to flow at nearly the same velocity of water.

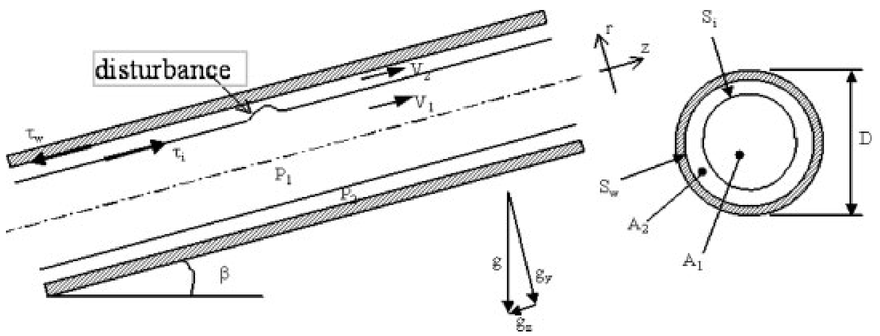

Figure 1. Schematic description of the core-annular flow pattern.

In this work, the development of a transition criterion for core-annular flow based on the linear stability analysis is presented. The equations were obtained through a one-dimensional approach, which allowed for the use of a rigorous two-fluid model. On one hand, the stability of the waves is studied assuming that a stationary core-annular flow is present, as in Joseph and coworkers. On the other hand, different interfacial waves are imposed and their effect on stability is also studied, as in Ooms and coworkers. Therefore, a combination of both classes of stability investigation is employed. The laminar–laminar flow is analyzed and fully resolved. Turbulence effects on the stability are assessed through experimentally obtained shape factors. The proposed general transition criterion includes in its formulation the IKH's discriminator. For all cases, the oil–water interface was considered smooth. Effects of interfacial tension, viscosity ratio, density difference, and shape factors on the stability of core-annular flow are analyzed in detail. The criterion predictions are then compared with recent data from the literature.

Hydrodynamic Stability and Transition Criteria for Core-Annular Flow

Figure 1 shows a schematic description of the core-annular flow pattern. The analysis is based on the two-fluid model, where the area-averaged unsteady one-dimensional differential equations of momentum and continuity are applied for the core and annulus phase. The equations are coupled through appropriated constitutive relations. The fluids are considered incompressible and the flow isothermal and without phase change. Index 1 is used for the core (oil) and index 2 for the annulus (water) phase.

The following balance equations can be formulated.

Continuity

$$\frac{\partial \varepsilon_1}{\partial t} + \varepsilon_1 \frac{\partial V_1}{\partial z} + V_1 \frac{\partial \varepsilon_1}{\partial z} = 0 \tag{1}$$

and

$$-\frac{\partial \varepsilon_1}{\partial t} + (1-\varepsilon_1)\frac{\partial V_2}{\partial z} - V_2 \frac{\partial \varepsilon_1}{\partial z} = 0 \tag{2}$$

Momentum

$$\rho_1 \left[\frac{\partial}{\partial t}(\varepsilon_1 V_1) + \frac{\partial}{\partial z}(\varepsilon_1 k_1 V_1^2)\right] + \varepsilon_1 \frac{\partial P_1}{\partial z} = -\frac{S_i \tau_i}{A} + \varepsilon_1 \rho_1 g_z \tag{3}$$

and

$$\rho_2 \left[\frac{\partial}{\partial t}(\varepsilon_2 V_2) + \frac{\partial}{\partial z}(\varepsilon_2 k_2 V_2^2)\right] + \varepsilon_2 \frac{\partial P_2}{\partial z} = \frac{S_i \tau_i}{A} - \frac{4}{D}\tau_w + \varepsilon_2 \rho_2 g_z, \tag{4}$$

where V corresponds to the average axial velocities, ρ corresponds to the densities, D is the internal diameter of the pipe, g_z is the gravitational component in the axial direction, ε_1 is the oil volumetric fraction or holdup, τ_i and τ_w are, respectively, the interfacial and pipe wall shear stresses and $\varepsilon_2 = (1 - \varepsilon_1)$, $S_i = \pi D(\varepsilon_1)^{0,5}$, $A = \pi D^2/4$, and $k = \langle V^2\rangle/\langle V\rangle^2$, keeping in mind that, $\langle f_n\rangle = 1/A_n \int f_n dA$, where f corresponds to any phase function and n is the phase index. The kinetic energy distribution parameters or shape factors, k_n, are given by

$$k_1 = \frac{\langle w_1^2\rangle}{V_1^2} = \frac{\frac{1}{\pi r^2}\int_0^{R_1} w_1^2 d(\pi r^2)}{\left[\frac{1}{\pi r^2}\int_0^{R_1} w_1 d(\pi r^2)\right]^2} \tag{5}$$

and

$$k_2 = \frac{\langle w_2^2\rangle}{V_2^2} = \frac{\frac{1}{\pi r^2}\int_{R_1}^{D/2} w_2^2 d(\pi r^2)}{\left[\frac{1}{\pi r^2}\int_{R_1}^{D/2} w_2 d(\pi r^2)\right]^2}, \tag{6}$$

where w_n corresponds to the velocity profiles and $R_1 = D\sqrt{\varepsilon_1}/2$.

A constitutive relation is necessary to close the problem. The Laplace-Young law is chosen in order to relate the pressures in the phases through the interfacial tension. The following relation is proposed:

$$P_1 - P_2 \frac{\sigma}{R_1}\left\{\frac{\partial}{\partial R_1}\left[R_1\left(1+\left\{\frac{\partial R_1}{\partial z}\right\}^2\right)^{-1/2}\right] - \frac{0.166\Delta\rho g_y \varepsilon_1 A}{\sigma}\right\}, \tag{7}$$

where the first term of the right-hand side of the above equation corresponds to the effect of variations on the core curvature in the axial direction (interfacial waves), the second term corresponds to the distortion of the core contour due to the density difference, which was described by Bannwart, σ is the oil–water interfacial tension and g_y is the gravitational component in the radial direction.

After making the adequate arrangements, the momentum equation for the annulus phase (Eq. 4) was subtracted from the momentum equation for the core phase (Eq. 3) and, in order to eliminate the pressure difference term, Eq. 7 is developed and substituted into the resulting equation. Therefore, the following equation, Eq. 8, is the area-averaged one-dimensional momentum equation for core-annular flow and together with Eqs. 1 and 2 forms a set of three equations involving the three unknown variables, V_1, V_2, and ε_1:

$$\begin{aligned}(1-\varepsilon_1)\rho_1&\left[\frac{\partial}{\partial t}(\varepsilon_1 V_1)+\frac{\partial}{\partial t}(\varepsilon_1 k_1 V_1^2)\right]\\&-\varepsilon_1\rho_2\left\{\frac{\partial}{\partial t}[(1-\varepsilon_1)V_2]+\frac{\partial}{\partial z}[(1-\varepsilon_1)k_2V_2^2]\right\}\\&+\varepsilon_1(1-\varepsilon_1)\frac{\partial}{\partial z}\left[\frac{2\sigma}{D\sqrt{\varepsilon_1}}+\frac{D\sigma}{16\varepsilon_1^{\frac{3}{2}}}\left(\frac{\partial\varepsilon_1}{\partial z}\right)^2\right.\\&\left.-\frac{D\sigma}{4\sqrt{\varepsilon_1}}\frac{\partial^2\varepsilon_1}{\partial z^2}-0.26\Delta\rho g_y D\sqrt{\varepsilon_1}\right]\\&=-(1-\varepsilon_1)\frac{Si}{A}\tau_i-\varepsilon_1\frac{Si}{A}\tau_i+\varepsilon_1\frac{4}{D}\tau_w\\&\quad+\varepsilon_1(1-\varepsilon_1)g_z(\rho_1-\rho_2)\end{aligned} \tag{8}$$

The hydrodynamic stability analysis comprises the study of the eventual increase of a small disturbance in the initial flow configuration, so that:

$$V_1=V_1^0+w_1;\quad V_2=V_2^0+w_2\quad\text{and}\quad\varepsilon_1=\varepsilon_1^0+\eta, \tag{9}$$

where superscript "0" indicates the equilibrium condition (without disturbance) and w_1, w_2, ε, η represent the small disturbances imposed on the velocities and core holdup, respectively. Furthermore, V_1^0, V_2^0, and ε_1^0 are constants and w_1, w_2, and $\eta = f\,(z, t)$.

According to the linear stability theory and following the method of small disturbances, w_1, w_2, and η are considered to be small and, therefore, second-order or higher terms can be neglected.[28] Such a simplification seems to be quite reasonable for the heavy oil–water annular flow, since the experimentally observed interfacial waves are relatively long and do not present a nonlinear behavior. Substituting Eq. 9 into Eqs. 1, 2, and 8 and neglecting all higher-order terms, the following continuity and momentum equations, respectively, arise:

$$\frac{\partial\eta}{\partial t}+\varepsilon_1^0\frac{\partial w_1}{\partial z}+V_1^0\frac{\partial\eta}{\partial z}=0 \tag{10}$$

$$-\frac{\partial\eta}{\partial t}+(1-\varepsilon_1^0)\frac{\partial w_2}{\partial z}-V_2^0\frac{\partial\eta}{\partial z}=0 \tag{11}$$

and

$$\begin{aligned}&\varepsilon_1^0(1-\varepsilon_1^0)\left(\rho_1\frac{\partial w_1}{\partial t}+\rho_2\frac{\partial w_2}{\partial t}\right)\\&+\left[\rho_1V_1^0(1-\varepsilon_1^0)(1-2k_1^0)+\rho_2V_2^0\varepsilon_1^0(1-2k_2^0)\right]\frac{\partial\eta}{\partial t}\\&-\left\{\begin{aligned}&\rho_1V_1^{0^2}(1-\varepsilon_1^0)\left(k_1^0-\varepsilon_1^0\left.\frac{dk_1}{d\varepsilon_1}\right|_0\right)\\&+\rho_2V_1^{0^2}\varepsilon_1^0\left[k_2^0-(1-\varepsilon_1^0)\left.\frac{dk_2}{d\varepsilon_1}\right|_0\right]\\&+\frac{(1-\varepsilon_1^0)\sigma}{D\sqrt{\varepsilon_1^0}}+0.13\Delta\rho g_yD\sqrt{\varepsilon_1^0}(1-\varepsilon_1^0)\end{aligned}\right\}\frac{\partial\eta}{\partial z}\\&-\frac{(1-\varepsilon_1^0)\sqrt{\varepsilon_1^0}D\sigma}{4}\frac{\partial^3\eta}{\partial z^3}=\left.\frac{\partial fe}{\partial V_1}\right|_0w_1+\left.\frac{\partial fe}{\partial V_2}\right|_0w_2+\left.\frac{\partial fe}{\partial\varepsilon_1}\right|_0\eta,\end{aligned} \tag{12}$$

where the right-hand terms of Eq. 8 were linearized through Taylor series expansion until the first order and

$$\text{fe}=f(V_1,V_2,\varepsilon_1)=-\frac{\text{Si}\tau_i}{A}+\varepsilon_1(1-\varepsilon_1)g_z(\rho_1-\rho_2)+\frac{4\varepsilon_1\tau_w}{D} \tag{13}$$

To eliminate the velocity disturbance terms, w_1 and w_2, Eqs. 10 and 11 are used and the derivate with respect to z of Eq. 12 is taken. Finally, the differential equation for the disturbance or stability equation for core-annular flow is achieved:

$$M\frac{\partial^4\eta}{\partial z^4}+N\frac{\partial^2\eta}{\partial t^2}+2E\frac{\partial^2\eta}{\partial t\partial z}+F\frac{\partial^2\eta}{\partial z^2}+G\left(\frac{\partial\eta}{\partial t}+H\frac{\partial\eta}{\partial z}\right)=0, \tag{14}$$

whose coefficients are

$$M=\frac{\sigma D\sqrt{\varepsilon_1^0}(1-\varepsilon_1^0)}{4};\quad N=\rho_1(1-\varepsilon_1^0)+\rho_2\varepsilon_1^0;$$

$$E=\rho_1V_1^0(1-\varepsilon_1^0)k_1^0+\rho_2V_2^0\varepsilon_1^0k_2^0;$$

$$\begin{aligned}F=&\rho_1V_1^{0^2}(1-\varepsilon_1^0)\left(k_1^0-\varepsilon_1^0\left.\frac{dk_1}{d\varepsilon_1}\right|_0\right)\\&+\rho_2V_2^{0^2}\varepsilon_1^0\left(k_2^0-(1-\varepsilon_1^0)\left.\frac{dk_2}{d\varepsilon_1}\right|_0\right)\\&+\frac{(1-\varepsilon_1^0)\sigma}{D\sqrt{\varepsilon_1^0}}+0.13\Delta\rho g_yD\sqrt{\varepsilon_1^0}(1-\varepsilon_1^0)\end{aligned}$$

$$G=-\frac{1}{\varepsilon_1^0}\left.\frac{\partial fe}{\partial V_1}\right|_0+\frac{1}{1-\varepsilon_1^0}\left.\frac{\partial fe}{\partial V_2}\right|_0,$$

$$H=\frac{-\frac{V_1^0}{\varepsilon_1^0}\left.\frac{\partial fe}{\partial V_1}\right|_0+\frac{V_2^0}{1-\varepsilon_1^0}\left.\frac{\partial fe}{\partial V_2}\right|_0+\left.\frac{\partial fe}{\partial\varepsilon_1}\right|_0}{-\frac{1}{\varepsilon_1^0}\left.\frac{\partial fe}{\partial V_1}\right|_0+\frac{1}{1-\varepsilon_1^0}\left.\frac{\partial fe}{\partial V_2}\right|_0}=c_0,$$

where c_o is the kinematic or continuity wave velocity.[29]

The exact solution of the linearized problem was developed according to the solution methodology presented by Whitham.[30] The normal mode was used to analyze the ondulatory character of Eq. 14, so that

$$\eta(z,t)=\eta_{\max}e^{i\mathbf{k}(z-ct)}, \tag{15}$$

where $\mathbf{k} = 2\pi/\lambda$ is the wave number, λ is the interfacial wavelength and c is the wave velocity.

Substituting Eq. 15 into Eq. 14, the following equation arises:

$$\mathbf{k}\frac{N}{G}\left(c^2-2\frac{E}{N}c+\frac{F-M\mathbf{k}^2}{N}\right)+t(c-H)=0 \tag{16}$$

Rearranging Eq. 16 and using the Viète's theorem, it becomes

$$\mathbf{k}\frac{N}{G}(c-c_1)(c-c_2)+t(c-c_0)=0, \tag{17}$$

where H was substituted by c_0. Approaching $c = c_1$ and $c = c_2$, Eq. 17 gives, respectively:

$$w \cong \mathbf{k}c_1 - i\frac{G}{N}\frac{c_1 - c_0}{c_1 - c_2} \tag{18}$$

and

$$w \cong \mathbf{k}c_2 - i\frac{G}{N}\frac{c_2 - c_0}{c_2 - c_1} \tag{19}$$

Following a temporal analysis, a necessary condition for the stability is that the imaginary component of w (Eqs. 18 and 19) must be negative, where w is the wave frequency. Rearranging Eqs. 18 and 19 and assuming that $c_1 > c_2$, the following criteria arise:

$$\begin{cases} 1 - G > 0 \\ 2 - c_1 \text{ and } c_2 \text{ must be real (Kelvin-Helmoholtz analysis)} \\ 3 - c_2 \le c_0 \le c_1 \end{cases} \tag{20}$$

The condition that c_1 and c_2 are real corresponds to the Kelvin-Helmholtz's stability criterion and can be expressed as[31]

$$V_0^2 + \frac{M\mathbf{k}^2 - F}{N} \ge 0, \tag{21}$$

where V_0 is the weighted average velocity, which corresponds to E/N.[29] Equation 21 is in general satisfied for relatively short wavelengths through the interfacial tension.

The criteria shown above (Eq. 20) can be combined into one single criterion. Solving Eq. 16 for c (real component) and applying the criteria, the following general stability criterion results:

$$0 \le \frac{\left(\frac{c_0}{V_0} - 1\right)^2}{1 + \frac{M\left(\frac{2\pi}{\lambda}\right)^2 - F}{NV_0^2}} \le 1 \tag{22}$$

It is worthwhile to stress that the denominator of the expression shown above can be recognized as the Kelvin-Helmholtz's discriminator, Eq. 21. Therefore, when the above general criterion value is negative, it means that the Kelvin-Helmholtz's criterion is being violated, whereas if the criterion value is above the unity, it means that the third criterion shown in Eq. 20 is not being satisfied. The validity of the presented criterion is tested using recent experimental data from the literature.

Application of the Transition Criterion and Comparison with Experimental Data

The proposed transition criterion is applied for a fully developed laminar–laminar annular flow with a smooth and concentric interface, also known as perfect core-annular flow (PCAF), whose exact solution of the Navier-Stokes equation led to the classic parabolic profiles.[14,32] The inviscid Kelvin-Hermholtz and general transition criteria are tested and the effect of turbulence on the stability is analyzed through adjusted shape factors.

Kelvin-Helmholtz's criterion (K-H)

In this case, coefficient G is negligible, so that the magnitude of the viscous effects can be ignored. Equation 9 becomes

$$M\frac{\partial^4 \eta}{\partial z^4} + N\frac{\partial^2 \eta}{\partial t^2} + 2E\frac{\partial^2 \eta}{\partial t \partial z} + F\frac{\partial^2 \eta}{\partial z^2} = 0 \tag{23}$$

Substituting Eq. 15 into Eq. 23, applying the essential requirement of the Kelvin-Helmholtz's criterion[31] to its dispersion relation and assuming that the velocity profiles are uniform ($k_1 = k_2 = 1$), the following stability criterion is obtained for core-annular flow:

$$\underbrace{(V_1^0 - V_2^0)^2}_{\text{destablizing}} < \left(\frac{1 - \varepsilon_1^0}{\rho_2} + \frac{\varepsilon_1^0}{\rho_1}\right) \times \left(\underbrace{\frac{\sigma D \pi^2}{\sqrt{\varepsilon_1^0}}\frac{1}{\lambda^2}}_{\text{stabilizing}} - \underbrace{\frac{\sigma}{D\varepsilon_1^{0^{3/2}}}}_{\text{destabilizing}} - \underbrace{\frac{0.13\Delta\rho g_y D}{\sqrt{\varepsilon_1^0}}}_{\text{destabilizing}}\right) \tag{24}$$

Analyzing the criteria shown earlier (Eq. 24), it becomes clear that the only stabilizing term is the one that contains the wavelength. Therefore, when viscous effects are negligible, the core-annular flow stability depends on the existence of the interfacial waves. This effect was already determined by Ooms; however, we suggest that the waves should give an additional "rigidity" to the core because of the curvature and magnitude of the interfacial tension. The role played by the interfacial tension in order to assure the adequate interface curvature was demonstrated by Bannwart, who concluded that for small Eötvös numbers the cross-section interface curvature is almost circular.

Equation 24 can also be rearranged in the following way:

$$\text{E0}^* < 24\left[\frac{\varepsilon_1^0 D^2}{\lambda^2} - \frac{1}{\pi} - 2We_2\left(\frac{s_0 - 1}{1 - \varepsilon_1^0}\right)^2 \frac{\varepsilon_1^{0^{3/2}}\rho_1}{\rho_1(1 - \varepsilon_1^0) + \rho_2\varepsilon_1^0}\right], \tag{25}$$

where

$$\text{E0}^* = \frac{(\rho_1 - \rho_2) g_y \varepsilon_1^0 A}{\sigma} \tag{26}$$

and

$$We_2 = \frac{\rho_2 J_2^2 D}{2\sigma} \tag{27}$$

are the Eötvös number and the Weber number of the annulus phase, respectively, and $s_0 = V_1^0/V_2^0$ is the holdup ratio. If it is assumed that there is no slippage between the phases, Eq. 25 becomes:

$$\text{E0}^* < 60\left(\frac{\varepsilon_1^0 D^2}{\lambda^2} - \frac{1}{\pi^2}\right) \tag{28}$$

If the existence of a maximum interfacial wavelength is considered as a necessary condition for stability, Eq. 28 can provide a stability criterion in terms of the Eötvös number quite similar to the one presented by Bannwart.

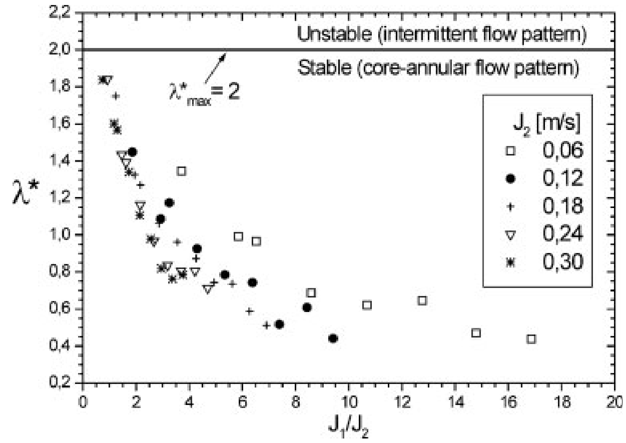

Figure 2. Wavelengths experimentally obtained for a stable upward vertical core-annular flow in function of the oil–water ratio[8] in comparison to the maximum wavelength predicted by the Kelvin-Helmholtz's transition criterion ($\lambda^*_{max} = 2$).

Maximum Interfacial Wavelength. For the simplest case of upward-vertical core-annular flow, the criterion shown above can be simplified even further, and Eq. 28 becomes

$$\lambda^* < \pi\sqrt{\varepsilon_1}, \tag{29}$$

where $\lambda^* = \lambda/D$ and the notation related to the equilibrium condition was dropped for convenience.

The transition criterion shown above is compared with the experimental data of Rodriguez and Bannwart[8] for upward-vertical core-annular flow. Figure 2 shows the observed wavelengths in function of the oil–water ratio for several water superficial velocities. The dots in Figure 2, which represent the experimental data, correspond to all situations where the core-annular flow pattern was observed. According to Rodriguez and Bannwart, the experimentally obtained minimum oil holdup, which is related to the beginning of the transition from Annular to Intermittent flow pattern (refer to Bannwart et al.,[2] for a detailed description of heavy oil–water flow patterns), was $\varepsilon_{1,\min} = 0.42$. By substituting $\varepsilon_{1,\min}$ into Eq. 29 the maximum dimensionless wavelength, $\lambda^*_{\max}$, can be calculated.

According to Eq. 29 and for the tested case, there is stable core-annular flow for $\lambda^* \leq \lambda^*_{\max} = 2$. The maximum dimensionless wavelength experimentally observed was $\lambda^* = 1.86 < \lambda^*_{\max}$, which is fairly in agreement with the theoretical result. As seem in Figure 2, the Kelvin-Helmholtz's criterion was able to predict quite well the region of stability for the upward vertical core-annular flow studied by Rodriguez and Bannwart.

The same good results were obtained for the horizontal core-annular flow. In this case, the criterion becomes:

$$\lambda^* < \frac{\pi\sqrt{\varepsilon_1}}{\sqrt{1 + 1.3\dfrac{\Delta\rho D^2 \varepsilon_1}{\sigma}}} \tag{30}$$

The criterion shown above predicts a maximum wavelength $\lambda^*_{\max} = 1.7$ (taking the experimentally observed minimum oil holdup $\varepsilon_{1,\min} = 0.75$ of Obregon-Vara[4]), which is in good agreement with the experimental data of Obregon Vara. This author observed a maximum dimensionless wavelength $\lambda^* = 1.13$ and 0.90 at the bottom and at the top of the pipe, respectively. The theoretical result is also compatible with the wavelengths observed by Oliemans, $\lambda^* = 0.7$.

Shape Factors. The velocity profiles of the phases can have an important effect on the stability of the core-annular flow pattern. The need for taking into account these parameters was recently pointed out by Song and Ishii[33] for gas–liquid two-phase flow. The effect of velocity distribution across the pipe is not usually considered and the shape factors are assumed to be equal to unity in situations where the fluids are considered inviscid.[31] However, in a real core-annular flow the shape factors can be significantly different from the unity because of viscous effects. Figure 3 shows the effect of k_2 on the Kelvin-Helmholtz's criterion, Eq. 21, applied to the core-annular flow with fluids of equal densities and smooth interface, neglecting the effect of interfacial tension ($We_2 \to \infty$.)

In Figure 3, the criterion indicates unconditional instability if $k_2 = 1$, but it predicts stability if $k_2 = 4/3$, which corresponds to a laminar flow in the annulus. This result clearly indicates that the velocity distributions have a stabilizing effect. This effect can be explained by the boundary-layer theory of laminar flows, whose stability is related to the shape of the velocity profile, originating the so-called Rayleigh's instability.[34]

General stability criterion

To apply the general transition criterion, Eq. 22, equations that describe the oil–water interfacial shear stress and pipe wall shear stress are necessary. For the laminar–laminar flow:

$$\tau_i = \frac{8\mu_1}{D\sqrt{\varepsilon_1}}\left\{V_1 - \left[s_0 V_2 - \frac{(\rho_1 - \rho_2)g_z D^2}{16\mu_2}\varepsilon_1\left(2 + \frac{1+\varepsilon_1}{1-\varepsilon_1}\right)\mathrm{Ln}\varepsilon_1\right]\right\} \tag{31}$$

$$\tau_w = \frac{8}{D(1-\varepsilon_1)}\left[V_2 - \frac{1}{4}(\rho_1 - \rho_2)g_z D\varepsilon_1\left(1 + \varepsilon_1 + \frac{2\varepsilon_1 \mathrm{Ln}\varepsilon_1}{1-\varepsilon_1}\right)\right] \tag{32}$$

where μ_1 and μ_2 are the oil and water viscosities, respectively.

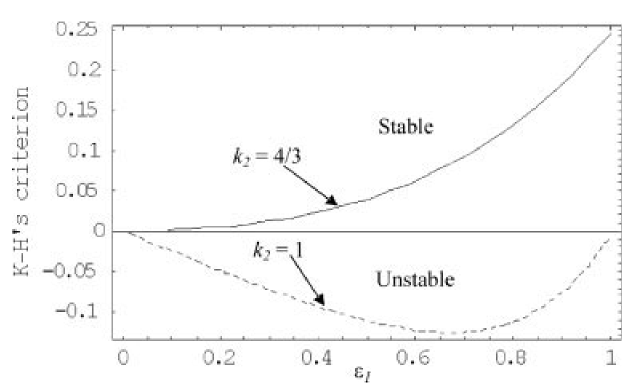

Figure 3. Effect of k_2 on the Kelvin-Helmholtz's stability criterion for core-annular flow ($k_1 = 1$ and $We_2 \to \infty$).

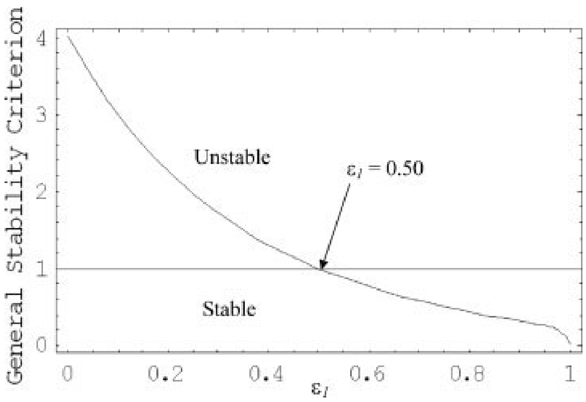

Figure 4. Stability of the perfect core annular flow (*We*$_2$ = 500 and *m* = 0.002).

The shape factors (Eqs. 5 and 6) were obtained from the velocity profiles deduced for a perfect core-annular flow:

$$w_1(r) = \frac{(G+\rho_1 g_z)(R_1^2 - r^2)}{4\mu_1} + \frac{(G+\rho_2 g_z)(R^2 - R_1^2)}{4\mu_2} + \frac{(\rho_1-\rho_2) g_z R_1^2 \mathrm{Ln}\left(\frac{R}{R_1}\right)}{2\mu_2}$$

$$w_2(r) = \frac{(G+\rho_2 g_z)(R^2 - r^2)}{4\mu_2} + \frac{(\rho_1-\rho_2) g_z R_1^2 \mathrm{Ln}\left(\frac{R}{r}\right)}{2\mu_2} \qquad (33)$$

where G corresponds to the pressure differential plus the gravitational term ($G = \rho_n g_z - \Delta P$) and $R = D/2$. For a laminar–laminar flow and fluids of equal densities, $k_1 = 1$ and $k_2 = 4/3$.

The general criterion was applied for fluids of equal densities. A relatively high Weber number ($We_2 = 500$) was taken, so that the interfacial tension effect was almost negligible. Figure 4 shows the discriminator defined by Eq. 22 as a function of oil holdup for an annulus-core viscosity ratio $m = 0.002$.

The criterion is satisfied for $\varepsilon_1 > 0.5$, Figure 4. Although it is related to the fully developed laminar–laminar annular flow with a smooth interface, the stability condition is in good agreement with the occurrence range experimentally observed by Rodriguez and Bannwart for upward-vertical core-annular flow, $0.42 < \varepsilon_1 < 0.86$, and Obregon-Vara for horizontal core-annular flow, $0.75 < \varepsilon_1 < 0.95$.

Effect of the Interfacial Tension on the Stability. The effect of the interfacial tension on the stability of the core-annular pattern depends on the wave length: short wavelengths tend to stabilize the flow, whereas long wavelengths tend to destabilize it. This effect was already determined by Ooms; however, the present one-dimensional analysis indicates the existence of a critical wavelength, for which there is a balance between interfacial tension forces and gravitational forces. Figure 5 shows the effect of the wavelength on the stability of the laminar–laminar core-annular flow of equal densities for two different Weber numbers, (Eq. 27).

For higher We_2, Figure 5a, there is a small or even none effect of the interfacial tension on the stability region ($\varepsilon_1 > 0.5$), which remains basically the same of Figure 4. For lower We_2 and $\lambda^* < \lambda^*_{\max}$, Figure 5b, the flow tends to become unconditionally stable, showing the importance that the interfacial tension acquires in this case.

Effect of the Viscosity Ratio on the Stability. The effect of the viscosity ratio on the stability of the laminar–laminar core-annular flow of equal densities is illustrated in Figure 6, where stability is observed when the more viscous fluid is in the core ($m < 1$) and occupies more than half of the pipe's cross-section or when the more viscous fluid is in the annulus ($m > 1$) and occupies more than half of the pipe's cross-section. In other words, the more viscous fluid occupies always more than half of the pipe's cross section. This result agrees with the core-annular flow studied by Rodriguez and Bannwart, is consistent with the conclusions of Joseph et al. and can be considered quite significant for the adopted one-dimensional approach.

Evaluation of the General Stability Criterion: Shape Factors and Density Differences. To take a last validity test of the general stability criterion, all the parameters involved in Eq. 22, superficial velocities, holdup, wave speed, wavelength, and shape factors were experimentally determined for each experimental point offered by Rodriguez and Bannwart. In our experiments, the flow of the core phase (oil) could be considered laminar ($k_1 = 1$), however the annulus flow (water) was mostly turbulent, since the Reynolds number at the annulus, $Re_2 = \rho_2 J_2 D/\mu_2$, was found in the range of $1700 < Re_2 < 8500$. k_2 was calculated through the adjusted water average velocity profile. For each experimental point, exponent p, which defines the velocity profile, was obtained

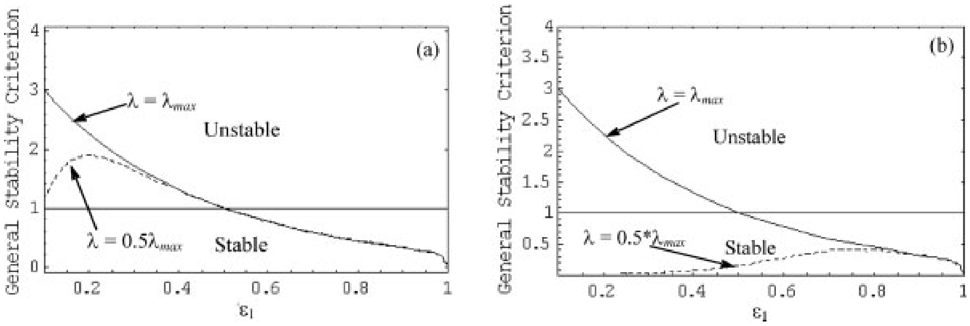

Figure 5. Effect of the wavelength on the stability of the laminar–laminar core flow of equal densities and *m* = 0.002; (a) *We*$_2$ = 500 and (b) *We*$_2$ = 0.5.

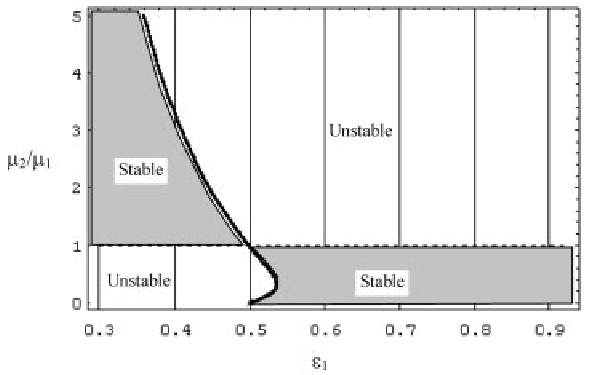

Figure 6. Effect of the viscosity ratio on the stability of the laminar–laminar core-annular flow of equal densities in function of the core phase holdup.

from the following expression: $w_2(y) = w_c(y/R)^{1/p}$, where $y = R - r$, $w_2(y)$ is the water velocity in function of y, w_c is the extrapolated velocity of the profile in the center of the pipe (notice that $w_2(R_1) = V_1$ and $R_1 = R\sqrt{\varepsilon_1}$) and R is the pipe radius. The experimental data were organized in such a way as to embrace all core-annular flow occurrence range observed by Rodriguez and Bannwart. The average water shape factor, from Eq. 6, was $k_2 = 1.15$. Figure 7 shows the general stability criterion discriminator as a function of the oil holdup, where the transition criterion is tested for each experimental point for the upward vertical core-annular flow studied by Rodriguez and Bannwart.

The criterion fails for points related to oil holdups lower than 0.50 (Figure 7). However, as reported in Maximum Interfacial Wavelength, such points would be related to the transition from annular to intermittent flow pattern. No improvement was observed in comparison with the criterion applied to a laminar–laminar core-annular flow of equal densities, which demonstrates that the density difference and the shape factors do not contribute to the stability in the studied case. The stability depends mainly on the magnitude of the interfacial tension and on the existence of the interfacial waves. The criterion was able to correctly predict the stability in more than 80% of the tested experimental points and no negative value was found, indicating that the Kelvin-Helmholtz's criterion was always satisfied.

The general stability criterion was also compared with experimental data for horizontal core flow. Table 1 shows the obtained results in comparison with the Obregon-Vara's experimental data, which were obtained in the same experimental facility described in this article and using a heavy crude oil of viscosity $\mu_1 = 1193$ mPa s and density $\rho_1 = 946$ kg/m^3.

Table 1 shows that only four points were wrongly predicted (in bold). Therefore, the criterion was also able to correctly predict the stability in about 80% of the tested experimental points for a horizontal core-annular flow. The authors argue that the wrongly predicted points were also placed in the vicinity of another subpatterns, therefore instabilities in the wavy annular pattern must have occurred.

Minimum Interfacial Wavelength and Curve of Neutral Stability. A minimum wavelength of half pipe diameter was experimentally observed in upward core-annular flow at high oil holdups by Rodriguez and Bannwart. In addition, those authors proposed an Eötvös number based on the wavelength, given by

$$E0_\lambda \cong \frac{\Delta\rho g\lambda D\sqrt{\varepsilon_1}}{4\sigma} \cong \text{const} \tag{34}$$

They observed that the Eötvös number (Eq. 34) is nearly constant. For an oil holdup of 0.90, which is related to transition from stable to unstable core-annular flow (refer to Bannwart et al.,[2] for a detailed definition of heavy oil–water flow patterns and subpatterns), the Eötvös number was 2. Therefore, Eq. 34 yields a wavelength of half pipe diameter for the flow condition tested by those authors, which is in perfect agreement with their experimental observations. The general

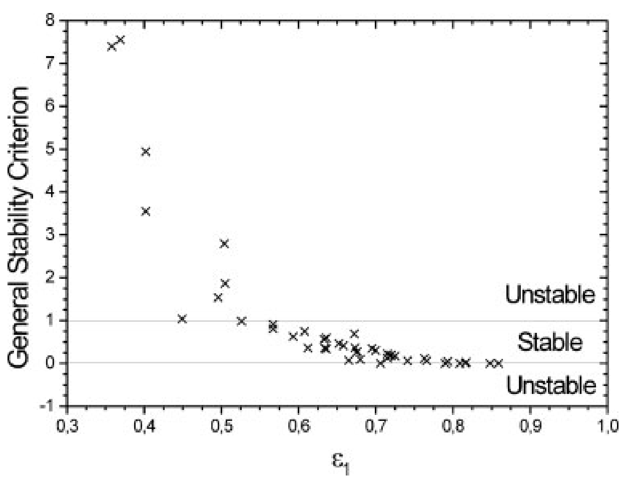

Figure 7. General stability criterion evaluation, in function of the oil holdup, for each experimental point and for upward vertical core-annular flow.

Table 1. General Stability Criterion Evaluation for each Obregon Vara's Experimental Point for Horizontal Core Flow

Test	J_2 (m/s)	J_1 (m/s)	Eq. 22
01	0.07	0.38	**2.64**
02	0.08	0.52	0.52
03	0.12	0.52	0.49
04	0.08	0.64	0.08
05	0.12	0.64	0.16
06	0.16	0.64	0.05
07	0.04	0.76	**1.03**
08	0.08	0.76	**1.35**
09	0.12	0.76	0.94
10	0.16	0.76	0.987
11	0.20	0.76	**1.19**
12	0.12	0.88	0.476
13	0.16	0.88	0.49
14	0.20	0.88	0.42
15	0.24	0.88	0.25
16	0.16	1.01	1.5×10^{-6}
17	0.20	1.01	6.3×10^{-3}
18	0.24	1.01	4.1×10^{-3}

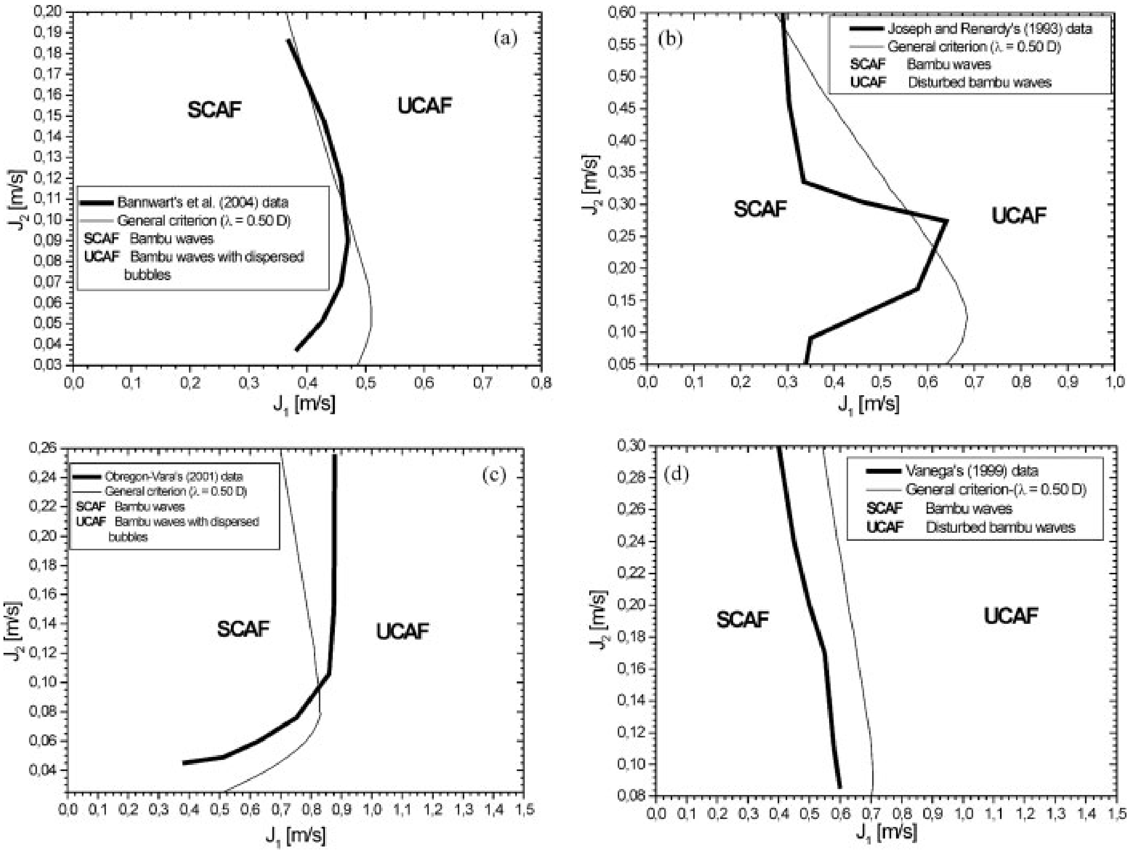

Figure 8. Transition boundary predicted by Eq. 22 (thin line, $\lambda = 0.5D$) and experimental transition boundary (thick line) observed by: (a) Bannwart et al.[6] (upward vertical core-annular flow, $D = 0.0284$ m, glass pipe, tap water, and crude oil, $\rho_1 = 935$ kg/m^3, $\mu_1 = 500$ mPa s, $\sigma = 0.029$ N/m); (b) Joseph and Renardy[32] (upward vertical core-annular flow, $D = 0.009525$ m, plastic pipe, tap water, and mineral oil, $\rho_1 = 881$ kg/m^3, $\mu_1 = 1300$ mPa s, $\sigma = 0.020$ N/m); (c) Obregon-Vara[4] (horizontal core-annular flow, $D = 0.0284$ m, glass pipe, tap water, and crude oil, $\rho_1 = 945$ kg/m^3, $\mu_1 = 1193$ mPa s, $\sigma = 0.029$ N/m) and (d) Vanegas-Prada[3] (horizontal core-annular flow, $D = 0.0276$ m, galvanized-steel pipe, tap water, and mineral oil, $\rho_1 = 960$ kg/m^3, $\mu_1 = 15{,}000$ mPa s, $\sigma = 0.020$ N/m).

stability criterion (Eq. 22) is applied with a wavelength $\lambda^* = 0.5$ and its validity is tested using experimental data from the literature.

On the flow maps of the oil and water superficial velocities (J_1 and J_2, respectively) in Figure 8, it is possible to observe a comparison between the experimental transition boundary (thick solid line) and the transition boundary predicted by Eq. 22 (thin solid line). In the same figure, SCAF stands for stable core-annular flow with bamboo waves and UCAF means unstable core-annular flow with either dispersed bubbles in the water annulus or disturbed interfacial waves.

Figure 8a shows the upward-vertical core-annular flow case studied by Bannwart et al.[2] The agreement between data and predictions is excellent, quantitatively and qualitatively. Notice that the trend of shrinkage of the stable area at low water superficial velocities is captured by the model. The vertical core-annular flow data of Joseph and Renardy are compared with the predictions of the general transition criterion (Eq. 22) in Figure 8b. The agreement between data and predictions is quite good. Once again, the data show a trend towards the shrinkage of the stable region at low water superficial velocities, which is captured by the model. The data of Obregon-Vara of horizontal core-annular flow are compared with predictions of Eq. 22 in Figure 8c. Good results are also observed for horizontal flow. Quantitatively, the agreement between predictions and data is quite good. Besides, the observed trends were captured by the model, especially at low water superficial velocities. The horizontal core-annular flow data of Vanegas-Prada are compared with the predictions of the general transition criterion (Eq. 22) in Figure 8d. For that very viscous oil, the agreement between data and predictions is again quite good.

Neutral-Stability Wave Number. The temporal analysis of the hydrodynamic stability predicts the exponential increase in the kinetic energy of an initially small disturbance with time. In the context of core-annular flow, the waves on the oil–water interface are believed to play a crucial role in the balance of forces acting on the oil core.[15,35] We propose that the presence of interfacial waves of intermediate wavelength ($\lambda \cong 0.50D$) should add, through the interfacial tension, an additional "rigidity" to the interface, contributing to the sta-

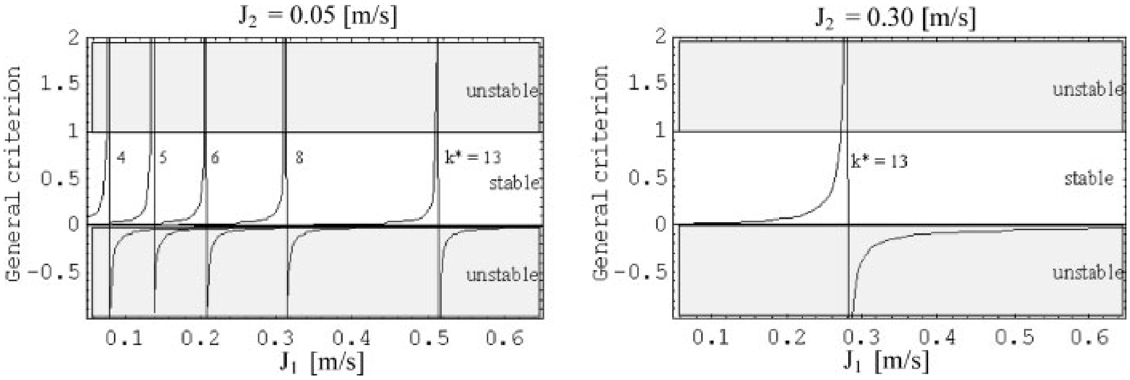

Figure 9. Effect of interfacial wave number ($k^* = \mathbf{k}D$) on the stability of upward vertical core-annular flow, based on the case of Bannwart et al.[6]; (a) J_2 = 0.05 m/s and (b) J_2 = 0.30 m/s.

bility for a given flow condition. Figure 9 shows, for the upward vertical core-annular flow studied by Bannwart et al.,[2] both regions of instability and the region of stability predicted by Eq. 22 and the effect of wave number on the stability criterion as a function of oil superficial velocity for two constant water superficial velocities. In Figure 9a, the lowest dimensionless wave number $k^* = 4(k^* = \mathbf{k}D)$ represents the last for which stability is still predicted for a constant $J_2 = 0.05$ m/s.

The inclusion of an even lower wave number into the interfacial tension term of Eq. 22 would imply no region of stability in Figure 9a, which would be in clear disagreement with the data of Bannwart et al.[6] The wave number that best agreed with the data was $k^* = 13$, which corresponds to the bamboo waves of wavelength $\lambda = 0.5D$ observed by Rodriguez and Bannwart. One can notice, by comparing Figures 9a,b, that the higher the water superficial velocity, the higher the limit wave number for which stability is still predicted by the stability criterion (Eq. 22). At $J_2 = 0.30$ m/s (Figure 8b), stable core-annular flow is predicted only if a limit wave number $k^* = 13$ is adopted, which corresponds to the wavelength of $\lambda = 0.5D$ observed in Rodriguez and Bannwart. In the region of higher water superficial velocities, higher water holdups as well as water annulus Reynolds numbers were observed by those authors. With the increase in the turbulence intensity, a stronger shear is expected to act on the interface, which promotes viscosity-induced disturbances that could lead to transition. The present theory proposes that interfacial tension forces should act against the destabilizing forces, keeping the oil core from breaking into dispersed bubbles. The results suggest that the observed wave of $k^* = 13$ adds to the interface the additional "rigidity" necessary for the stability of the core-annular flow pattern for a given flow condition.

Figure 10 shows, for the horizontal core-annular flow studied by Obregon-Vara, the effect of wave number on the stability criterion predictions (Eq. 22) as a function of oil superficial velocity for two constant water superficial velocities. Similar results are observed for horizontal core-annular flow. In Figure 10b, the lowest wave number $k^* = 6.0$ represents the last for which stability is still predicted by Eq. 22. The wave number that best agreed with Obregon-Vara's data was $k^* = 13$, which corresponds to the minimum interfacial wavelength observed by that author.

The results for horizontal and upward vertical flow strongly suggest that the interfacial tension term plays an indispensable role in the correct prediction of the stable region of core-annular flow pattern.

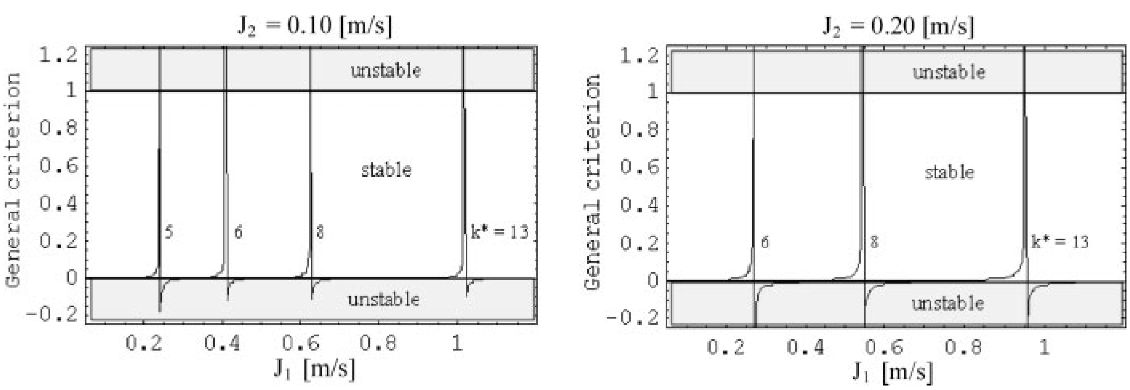

Figure 10. Effect of interfacial wave number ($k^* = \mathbf{k}D$) on the stability of the horizontal core-annular flow, based on the case of Obregon-Vara[4]; (a) J_2 = 0.10 m/s and (b) J_2 = 0.20 m/s.

Summary

A general transition criterion based on the linear stability theory and whose main contribution consists in the simplicity of the one-dimensional approach has been proposed. The interface is assumed to be smooth and the destabilizing buoyancy force due to a possible density difference between oil and water is incorporated for a non-vertical pipe. On one hand, the stability of the waves is studied assuming that a stationary core-annular flow is present, as in Joseph and coworkers. On the other hand, different interfacial waves are imposed and their effect on stability is also studied, as in Ooms and coworkers. Therefore, a combination of both classes of stability investigation is employed. The general criterion includes the Kelvin-Helmholtz's criterion as a particular case and allows for the analysis of the influence of several properties, such as densities, viscosities, interfacial tension, shape factors, and neutral stability wave number. When compared with several experimental data sets from the literature, the criterion was able to predict with very good accuracy the region of existence of the stable core-annular flow pattern for both upward vertical and horizontal flows.

Generally speaking, the developed criterion indicates the stability of the annular pattern when one or more of the following conditions are satisfied: (a) the core phase is more viscous than the annulus phase and occupies more than half of the cross section; (b) the buoyancy force in the core must be small in comparison to interfacial tension force (low Eötvös number); (c) the inertial force in the annulus must be small in relation to the interfacial tension force (low Weber number in the annulus); and (d) the annulus-phase velocity profile approaches to the parabolic profile ($k_2 = 4/3$ for a laminar flow in the annulus and $k_2 = 1.15$ for a typical turbulent case).

According to the proposed general stability model, the interfacial waves favor the core-annular flow stability. This effect increases as the wavelength decreases. If a theoretical maximum wavelength of twice the pipe diameter is considered as a necessary condition for stability, the IKH analysis provides a stability criterion in terms of the Eötvös number. When applying the general transition criterion, the best agreement with data was obtained with the inclusion of a theoretical minimum wavelength of half pipe diameter. According to recently published data, the maximum wavelength ($\lambda = 2D$) is associated with the transition from stable core-annular flow to intermittent flow and the minimum wavelength ($\lambda = D/2$) with the transition from stable to unstable core-annular flow with either dispersed bubbles in the water annulus or disturbed and nonlinear interfacial waves. The results strongly suggest that the interfacial tension term plays an indispensable role in the correct prediction of the stable region of core-annular flow pattern. The incorporation of waves of intermediate wavelength into the viscous transition model produced significantly better predictions in comparison with data from the literature for both upward vertical and horizontal core-annular flows, supporting the theory that waves should add an additional "rigidity" to the interface and delay the transition to intermittent or dispersed flows.

Acknowledgments

This work was supported by PETROBRAS, FINEP, CNPq, and FAPESP (proc. 04/13374-7). The authors are grateful to the Research and Development Center of PETROBRAS-CENPES for the donated material and technical assistance. Sincere acknowledgements are extended to Rosa Maribel Obregon Vara and Isabela Simões Wang for their assistance in the experiments. We are also indebted to Prof. René Oliemans for the support he provided during Oscar Rodriguez's visit to the Delft University in 2001.

Literature Cited

1. Briggs PJ, Baron RP, Fulleylove RJ, Wright MS. Development of heavy-oil reservoirs. *SPE*. 1998;15748:206–214.
2. Bannwart AC, Rodriguez OMH, de Carvalho CHM, Wang IS, Obregon-Vara RM. Flow patterns in heavy crude oil-water flow. *J Energy Resour Technol Trans ASME*. 2004;126:184–189.
3. Vanegas-Prada JW. Experimental study of core annular flow in the ultraviscous oil lift. M.Sc. Dissertation, The State University of Campinas, 1999.
4. Obregon-Vara RM. Hydrodynamics of the two-phase heavy oil–water flow in a horizontal pipe. M.Sc. Dissertation, The State University of Campinas, UNICAMP, 2001.
5. Rodriguez OMH. Interface shape and pressure gradient in upward core-annular flow. Ph.D. Thesis, The State University of Campinas, UNICAMP, 2002.
6. Bannwart AC, Rodriguez OMH, Trevisan FE, Vieira FF, Carvalho CHM. Flow patterns and pressure gradient in horizontal, upward inclined and vertical heavy oil–water–gas flows: experimental investigation and full-scale experiments. In 3rd International Symposium on Two-phase Flow Modelling and Experimentation, Pisa, 2004.
7. Bannwart AC, Vieira FF, Trevisan FE, Rodriguez OMH, de Carvalho CHM. Pressure drop in water-assisted three-phase flow of heavy oil and gas in a vertical pipe. In 5th North American Conference on Multiphase Technology–Transfering Technology, BHR Group Limited, 2006:421–433.
8. Rodriguez OMH, Bannwart AC. Experimental study on interfacial waves in vertical core flow. *J Pet Sci Eng*. 2006;54:140–148.
9. Rodriguez OMH, Bannwart AC. Analytical model for interfacial waves in vertical core flow. *J Pet Sci Eng*. 2006;54:173–182.
10. Charles ME, Govier GW, Hodgson GW. The horizontal pipeline flow of equal density oil–water mixtures. *Can J Chem Eng*. 1961;39:27–36.
11. Flores JG, Chen XT, Sarica C, Brill JP. Characterization of oil–water flow patterns in vertical and deviated wells. In: SPE Annual Technical Conference and Exhibition, SPE 38810, San Antonio, Texas, 1997.
12. Trallero JL. Oil–water flow patterns in horizontal pipes. PhD Thesis, The University of Tulsa, 1995.
13. Elseth G. An experimental study of oil–water flow in horizontal pipes. Ph.D. Thesis, The Norwegian University of Science and Technology, 2001.
14. Russel TWF, Charles ME. The effect of the less viscous liquid in the laminar flow of two-immiscible liquids. *Can J Chem Eng*. 1959; 37:18–24.
15. Ooms G. The hydrodynamic stability of core-annular flow of two ideal liquids. *Appl Sci Res*. 1972;26:147–158.
16. Ooms G, Segal A, Van der Wees AJ, Meerhoff R, Oliemans RVA. A theoretical model for core-annular flow of a very viscous oil core and a water annulus throught a horizontal pipe. *Int J Multiphase Flow*. 1984;10:41–60.
17. Oliemans RVA. The lubricating-film model for core-annular flow. Ph.D. Thesis, Delft University of Technology, 1986.
18. Joseph DD, Renardy Y, Renardy M. Instability of the flow of immiscible liquids with different viscosities in a pipe. *J Fluid Mech*. 1984; 141:309–317.
19. Feng J, Huang PY, Joseph DD. Dynamic simulation of the motion of capsules in pipelines. *J Fluid Mech*. 1995;289:233–245.
20. Howard H, Patankar N. Non-axisymmetric instability of core-annular flow. *J Fluid Mech*. 1995;290:213–224.
21. Bai R. Traveling waves in a high viscosity ratio and axisymmetric core annular flow. Ph.D. Thesis, University of Minnesota, 1995.
22. Bai R, Kelkar K, Joseph DD. Direct simulation of interfacial waves in a high-viscosity-ratio and axisymmetric core-annular flow. *J Fluid Mech*. 1996;327:1–34.
23. Bai R, Joseph DD. Steady flow and interfacial shapes of a highly viscous dispersed phase. *Int J Multiphase Flow*. 2000;26:1469–1491.

24. Brauner N, Maron DM. Classification of liquid–liquid two-phase flow systems and the prediction of flow pattern maps. In: 2nd International Symposium on Two-Phase Flow Modeling and Experimentation, ISTP'99, Pisa, 1999:747–754.
25. Bannwart AC. Bubble analogy and stabilization of core-annular flow. *J Energy Resour Technol.* 2001;123:127–132.
26. Barnea D. On the effect of viscosity on stability of stratified gas–liquid flow—application to flow pattern transition at various pipe inclinations. *Chem Eng Sci.* 1991;46:2123–2131.
27. Barnea D, Taitel Y. Kelvin-Helmholtz stability criteria for stratified flow: viscous versus non-viscous (inviscid) approaches. *Int J Multiphase Flow.* 1993;19:639–649.
28. Schlichting H. *Boundary-Layer Theory*. New York: McGraw-Hill, 1979.
29. Wallis GB. *One-Dimensional Two-Phase Flow*. New York: MacGraw-Hill, 1969.
30. Whitham GB. *Linear and Nonlinear Waves*. New York: Wiley, 1974.
31. Ishii M. Wave phenomena and two-phase flow instabilities. In: Hetsroni G, editor. *Handbook of Multiphase Systems*. New York: McGrw-Hill, 1982:2/95–2/98.
32. Joseph DD, Renardy Y. *Fundamentals of Two-Fluid Dynamics. II. Lubricated Transport, Drops and Miscible Liquids*. New York: Springer-Verlag, 1993.
33. Song JH, Ishii M. On the stability of a one-dimensional two-fluid model. *Nucl Eng Des.* 2001;204:101–115.
34. White FM. *Viscous Fluid Flow*. New York: Wiley, 1991.
35. Miesen RHM, Beijnon G, Duijvestijn PEM, Oliemans RVA, Verheggen TMM. Interfacial waves in core-annular flow. *J Fluid Mech.* 1992;238:97–117.

Manuscript received Aug. 17, 2007, and revision received Oct. 1, 2007.